Exercise 20

Given $f(x) = \frac{1}{x}$ and $g(x) = x - 3$, find the following:

(a) $(f \circ g)(x)$

(b) the domain of $(f \circ g)(x)$ in interval notation

(c) $(g \circ f)(x)$

(d) the domain of $(g \circ f)(x)$

(e) $\left(\frac{f}{g}\right)(x)$

Solution

Compute $(f \circ g)(x)$ by plugging the formula for $g(x)$ where x is in the formula for $f(x)$.

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= \frac{1}{(x-3)} \\ &= \frac{1}{x-3} \end{aligned}$$

It's impossible to divide by zero, so $x - 3 \neq 0$. Add 3 to both sides to solve for x: $x \neq 3$. Therefore, the domain of $(f \circ g)(x)$ in interval notation is $(-\infty, 3) \cup (3, \infty)$. Compute $(g \circ f)(x)$ by plugging the formula for $f(x)$ where x is in the formula for $g(x)$.

$$\begin{aligned} (g \circ f)(x) &= g(f(x)) \\ &= \left(\frac{1}{x}\right) - 3 \\ &= \frac{1}{x} - 3 \end{aligned}$$

It's impossible to divide by zero, so $x \neq 0$. Therefore, the domain of $(g \circ f)(x)$ in interval notation is $(-\infty, 0) \cup (0, \infty)$.

$$\begin{aligned} \left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)} \\ &= \frac{\frac{1}{x}}{x-3} \\ &= \frac{1}{x(x-3)} \end{aligned}$$

It's impossible to divide by zero, so

$$\begin{gathered} x(x-3) \neq 0 \\ x \neq 0 \quad \text{or} \quad x - 3 \neq 0 \\ x \neq 0 \quad \text{or} \quad x \neq 3. \end{gathered}$$

Therefore, the domain of $(f/g)(x)$ is $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$.